Response of Large Space Structures with Stiffness Control

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For large space structures such as the 100-m-diam wrap-rib deployable antenna and spinning solar sail whose out-of-plane stiffness is derived from in-plane tension, the out-of-plane motion can be actively controlled by time-varying in-plane tension. An elastic string is used to demonstrate the proposed approach, which results in a nonlinear ordinary differential equation. The approximation method is outlined from which the magnitude of time-varying tension can be determined based on the efficiency factor and the time factor.

Nomenclature

L = string length
t = time coordinate
T = tension force
T_0 = constant tension force
ΔT = time-varying tension force
$\tilde{T}$ = normalized tension force
U = time variable of Y
V = spatial variable of Y
x = axial coordinate
X = nondimensional axial coordinate
y = transverse displacement of string
Y = nondimensional transverse displacement of string
α = efficiency factor
β = time factor
γ = equivalent linear damping
ϵ = proportional constant
ρ = mass density per unit length of string
τ = nondimensional time coordinate
ω = fundamental transverse frequency of string

Introduction

RECENT trends in satellite technology have been towards significantly larger spacecraft. Space structures with large dimensions and minimal weight are necessarily flexible. Since the performance of these systems relies, in many cases, on precise pointing and/or a precise structural configuration, it is necessary to minimize their structural dynamic response to disturbances. The sources of these disturbances include meteoroid impact, docking, deployment, and maneuvers.

For large space structures, the increased flexibility and size require the control system to recognize and compensate for multiple elastic mode behavior. The problems of devising a properly stable control scheme, modeling the structure, and making in-flight response measurements using sensors, and the subsequent interpretation of these measured data have become increasingly difficult. These problems have received a great deal of attention in recent years. Typical relevant literature can be found in Ref. 1. One of the major objectives for the control of large space structures is to provide active damping for the transient responses. This is accomplished by making the control forces proportional to the velocities of the responding structures. These control forces are usually applied to the structures via actuators such as compressed gas jets and small rockets. For certain large structures, whose material density per unit area is extremely small, these control actuators become too bulky and heavy. It would be impractical to utilize a distributed system of actuators for these types of structures. Alternative actuation schemes become desirable.

For example, one of the large space structures currently under development is the wrap-rib mesh deployable antenna with a diameter of 100 m. This antenna mesh is attached to the rib skeleton in a pretensed condition sufficient to provide out-of-plane membrane stiffness (Ref. 2). The pointing accuracy is 1 μrad of the intended direction and the surface of the antenna must stay within 1 m of the intended configuration. Another example is the spinning sail (Ref. 3), which consists of a spacecraft propelled by photon light pressure impinging upon 12 blade-like surfaces which, when spun, provide stability controls. Each blade is about 8 m wide by 4000 m long, and is made of extra-light material capable of carrying tension only. The surface thicknesses for the antenna and the sail are 0.254×10^{-1} and 0.254×10^{-2} mm, respectively. Clearly, conventional actuators are not suitable for these.

One common feature of the two examples is the fact that the surfaces of the large space structures have little or no intrinsic out-of-plane stiffness. Bending and torsional stiffness are provided by the tensions between the ribs for the antenna and by the centrifugal force (Ref. 4) for the spinning sail. The motions due to disturbances are predominantly out-of-plane, and the stiffness to resist these disturbances comes from the tension in the in-plane direction. If the conventional actuators are not suitable to control the out-of-plane motion, the alternative is to control the in-plane tension.

Since the tension is associated with stiffness terms, instead of damping terms, there exists a fundamental question whether the out-of-plane motion can be controlled by varying the in-plane tension. Another question is whether control devices for the varying tension can be found which are also compatible with the light weight and the thin surfaces. The solution to the second question can be realized in the piezoelectric strain transducer, which generates a voltage when strain is produced in its designed usage. However, conversely, when a voltage is applied, a corresponding strain is produced in the transducer. This in turn generates loadings to the specimen to which the transducer is attached.

This type of device was studied briefly in the 1950's (Ref. 5), and was again used recently on an optical structure (Ref. 6) and an experimental demonstration (Ref. 7). The strain transducer is a compact rectangular strip which can be bonded to the test specimen. The transducers used in the experiment of Ref. 7 were Gulton SC-4 type, with dimensions of $25.4 \times 6.35 \times 0.254$ mm and weighing only 0.4 g. The test specimen was an omnidirectional antenna made of fiberglass with inertial mass at the top similar to that used in the Pioneer/Venus spacecraft. The total weight exceeded 1800 g and four transducers were used as control devices. Manufacturing technology has matured, and even smaller, thinner, and lighter strain transducers are being produced with semiconductor materials.

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It appears that these devices are ideally suited for varying the tension in large space structures, and they have a further advantage: no reaction mass such as compressed gas or fuel is required, as compared to conventional jets and rockets. Therefore, theoretically they can operate forever as long as the power supply is available.

The availability of suitable tension control devices is thus determined. The objective of the present investigation is to answer the fundamental question of whether in-plane tension varying can control the out-of-plane motion. Once this has been established, the question of how the tension should be varied to achieve most effective control will be studied.

Approach

One of the simplest components which derives its flexural rigidity from the axial tension is a string whose equation of motion is as follows:

$$\rho \frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x}\left(T \frac{\partial y}{\partial x}\right) \tag{1}$$

with the initial condition as $y = 0$, $\partial y/\partial t = \sin(\pi x/L)$, at $t = 0$, and the boundary condition as $y = 0$ at $x = 0, L$, where y = transverse displacement, x = axial coordinate, t = time coordinate, ρ = mass density per unit length of the string, T = tension in the string, and L = string length.

The tension T consists of two parts, a constant, T_0, and a time-varying, $\Delta T(t)$

$$T = T_0 + \Delta T(t) \tag{2}$$

The objective is to study how to control the time-varying tension ΔT in order to damp out the initial velocity.

Equation (1) will be nondimensionalized as

$$\frac{\partial^2 Y}{\partial \tau^2} = \frac{\tilde{T}}{\pi^2} \frac{\partial^2 Y}{\partial X^2} \tag{3}$$

where

$$Y = \frac{y}{L}, \quad X = \frac{x}{L}, \quad \tau = \omega_0 t$$

$$\omega_0 = \frac{\pi}{L}\sqrt{\frac{T_0}{\rho}}, \quad \tilde{T} = \frac{T}{T_0} \tag{4}$$

Equation (3) can be solved by separation of variables. Let

$$Y(X,\tau) = U(\tau)V(X) \tag{5}$$

Upon substitution of Eq. (5) into Eq. (3), and consideration of initial and boundary conditions, one obtains

$$V = \sin(\pi X)$$

$$\frac{d^2 U}{d\tau^2} + \tilde{T} U = 0$$

$$\frac{dU}{d\tau} = 1.0, \; U = 0 \quad \text{at} \quad \tau = 0 \tag{6}$$

The solution to Eq. (6) is dependent on the form of the coefficient $\tilde{T}$ which is the nondimensionalized tension. For the constant tension T_0 only, the coefficient $\tilde{T}$ is a unity and the solution is a harmonic function with constant magnitude. This corresponds to the usual string vibration due to initial velocity specified in Eq. (1). Without the presence of a damping term in the governing equation, the string vibrates harmonically without decay. Now the time-varying part of the tension ΔT is assigned a value based on the measurement from a sensor placed at the midspan of the string. For instance, if ΔT is to be proportional to the absolute value of velocity measurement, then

$$\Delta T = \epsilon \left| \left(\frac{T_0}{\omega_0 L} \frac{\partial y}{\partial t}\bigg|_{x=L/2} \right) \right| \tag{7}$$

where ϵ is the proportional constant which determines the magnitude of the tension ΔT and $T_0/\omega_0 L$ is a constant for dimensional purposes. Using Eqs. (4) and (7), the governing equation in Eq. (6) becomes:

$$\frac{d^2 U}{d\tau^2} + \epsilon \left| \frac{dU}{d\tau} \right| U + U = 0 \tag{8}$$

Equation (8) is a nonlinear, ordinary, second-order differential equation. The behavior of its solution can best be examined by the phase-plane diagram which plots the velocity against the displacement. Figure 1 is such a diagram for the proportional constant $\epsilon = -0.5$ and $\epsilon = -1.0$ cases. First, it is interesting to note that indeed the velocity amplitude is reduced after one cycle of motion. Therefore, the time-varying tension ΔT can reduce the initial velocity of the string and act as an active damping device. However, further examination indicates that this type of control is not very effective. The reasons are: first, for relatively large time-varying tension, $\epsilon = -0.5$ and $\epsilon = -1.0$, the reduction of velocity is not very significant, and second, it seems that the displacement becomes unacceptably large for large ϵ. It should be noted that for the case of constant tension only, the phase-plane diagram is a circle with a unity radius. This means that the displacement never exceeds 1.0. Figure 2 shows the velocity time history of the Eq. (8) solution. It is clear that the larger the ϵ, the more deviations of response there are from the simple harmonic motion. This indicates heavy participation of higher and/or subharmonic motions. Also from the fact that the period of the motion is increased with increasing ϵ, one may conclude that the nonlinear term representing the time-varying

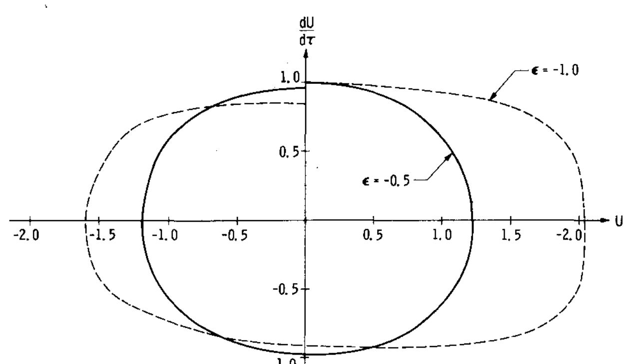

Fig. 1 Phase plane diagram for $\tilde{T} = 1 + \epsilon |dU/d\tau|_{X=\frac{1}{2}}$.

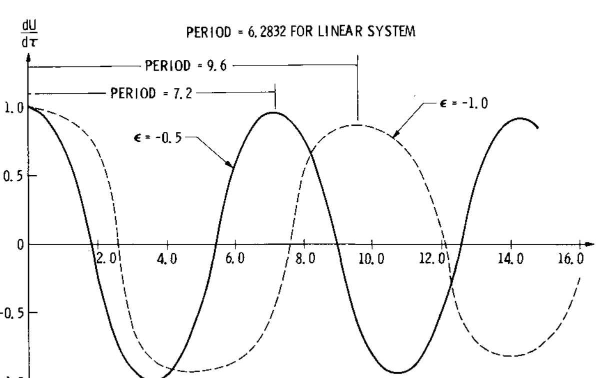

Fig. 2 Velocity time history for $\tilde{T} = 1 + \epsilon |dU/d\tau|_{X=\frac{1}{2}}$.

Table 1 Velocity reduction

Case No.	Tension ΔT	Differential equation	Velocity amplitude
1	$-0.2\left\lvert\left(\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right)\right\rvert$	$\frac{d^2U}{d\tau^2} - 0.2\left\lvert\frac{dU}{d\tau}\right\rvert U + U = 0$	0.970
2	$+0.2\left\lvert\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right\rvert$	$\frac{d^2U}{d\tau^2} + 0.2\left\lvert\frac{dU}{d\tau}\right\rvert U + U = 0$	1.024
3	$+0.2\left(\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right)$	$\frac{d^2U}{d\tau^2} + 0.2\frac{dU}{d\tau}U + U = 0$	0.997
4	$-0.2\left(\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right)$	$\frac{d^2U}{d\tau^2} - 0.2\frac{dU}{d\tau}U + U = 0$	0.997
5	$+0.2\left(\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right)\frac{y_{x=L/2}}{\lvert y_{x=L/2}\rvert}$	$\frac{d^2U}{d\tau^2} + 0.2\frac{dU}{d\tau}\lvert U\rvert + U = 0$	0.651
6	$-0.2\left(\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right)\frac{y_{x=L/2}}{\lvert y_{x=L/2}\rvert}$	$\frac{d^2U}{d\tau^2} - 0.2\frac{dU}{d\tau}\lvert U\rvert + U = 0$	2.124

tension contributes to the reduction of stiffness more than the contribution in damping.

To investigate a better way of tension control, the different forms of time-varying tension are compared. The form of time-varying tension, its corresponding differential equation, and the velocity amplitude at the end of two cycles of motion are listed in Table 1. In Cases 1 and 2, the time-varying tension ΔT is proportional to the minus and plus absolute value of velocity measurement, respectively. In Cases 3 and 4, the time-varying tension ΔT is proportional to the plus and minus velocity measurement, respectively. In Case 5, the magnitude of ΔT is proportional to the velocity measurement, but the sign is determined by the product of the velocity measurement and its corresponding displacement. In Case 6, the ΔT is that of Case 5 multiplied by -1. The results clearly show that Case 5 is the most effective way to control the tension for actively damping out the disturbances. Its equation is written as:

$$\frac{d^2U}{d\tau^2} + \epsilon\lvert U\rvert\frac{dU}{d\tau} + U = 0 \tag{9}$$

Although this is also a nonlinear equation, the nonlinear term does function as a damping term with variable damping coefficient but always positive. It is a very effective damping with velocity amplitude reduction of 35% after two cycles of motion. Figure 3 is the phase-plane diagram for Eq. (9). The velocity and the displacement are well-behaved and are very close to those of the linear case. Thus, an affirmative answer to the fundamental question of whether varying the in-plane tension can indeed control the out-of-plane motion due to initial disturbances has been realized. This most effective way of varying the tension among the cases studied is

$$\Delta T(t) = \left[\epsilon\left(\frac{T_0}{\omega_0 L}\frac{\partial y}{\partial t}\Big\rvert_{x=L/2}\right) y\rvert_{x=L/2}\right] \Big/ \lvert y_{x=L/2}\rvert \tag{10}$$

Equation (10) determines that the tension should be proportional to the velocity measurement, with the sign according to the product of velocity and displacement. However, the actual magnitude of the tension, in this case the constant ϵ, is undetermined. Intuitively, one wants to use large tension for rapid reduction of disturbance amplitude. However, there is also the efficiency consideration, namely, the larger tension, the more power required by the control device to generate the large tension. If the power supply to the large space structure is limited, which should be true in most cases, the efficiency of the system must be considered. On the other hand, it may be necessary, because of mission requirements, that responses from disturbances be damped out within a certain period.

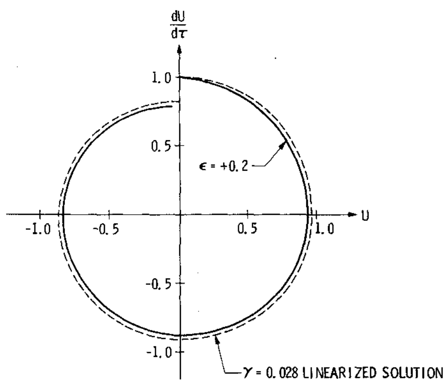

Fig. 3 Phase plane for $\tilde{T} = 1 + \epsilon\left(\left\lvert\frac{dU}{d\tau}\right\rvert \cdot U\right) \Big/ \lvert U\rvert$.

Then the contradicting factors, the efficiency vs dampout time, must be compromised to arrive at an optimal condition. This will be studied in the following section.

It should be emphasized that the cases studied in Table 1 are by no means all the possible control schemes. In fact, these cases are chosen for their straightforward simplicity instead of efficient control schemes. As mentioned before, the objective is to demonstrate that varying stiffness can achieve damping effects. A reviewer of this paper brought out the fact that the input to the control transducer as expressed by Eq. (10) is quadratic as compared to the quasilinear input required for the control scheme using $\lvert U\rvert\,\mathrm{sgn}(dU/d\tau)$. Since the maximum input voltage and maximum output force of a given transducer are limited, the quasilinear control has a much better range of control.

Optimal Control Consideration

Two aspects of the problem will be considered. One is the efficiency factor defined as the energy required to damp out a unit amplitude due to the disturbance; the other is the time factor defined as the time required to damp out a given amplitude due to the disturbances. Both factors will be calculated as a function of the time-varying tension magnitude coefficient. Since Eq. (9) is nonlinear, it is not obvious how the coefficient is affecting the solution. Hence, an equivalent linear equation will be sought. Equation (9) is rewritten as:

$$\frac{d^2U}{d\tau^2} + 2\gamma\frac{dU}{d\tau} + U + (\epsilon\lvert U\rvert - 2\gamma)\frac{dU}{d\tau} = 0 \tag{11}$$

where γ is the linear damping coefficient whose magnitude will be related to ϵ by way of an approximation technique, namely, the Galerkin's method. Let the nonlinearity in Eq. (11) be sufficiently small, such that the solution can be approximated by a linear solution

$$U(\tau) = e^{-\gamma t}\sin\left(\sqrt{1-\gamma^2}\,\tau\right) \tag{12}$$

Equation (12) satisfies the linear part of Eq. (11) and the initial conditions specified in Eq. (6). Upon substitution of Eq. (12) into Eq. (11), a residual error η will result:

$$\eta = \left[\epsilon\left|e^{-\gamma\tau}\sin\left(\sqrt{1-\gamma^2}\,\tau\right)\right| - 2\gamma\right] \times e^{-\gamma\tau}\left[\sqrt{1-\gamma^2}\cos\left(\sqrt{1-\gamma^2}\,\tau\right) - \gamma\sin\left(\sqrt{1-\gamma^2}\,\tau\right)\right] \tag{13}$$

Since only the linear damping coefficient γ is undetermined, the residual error η is minimized with respect to γ. The procedure calls for the results of the residual error η multiplied by a weighting function and integrated over one cycle to vanish. The weighting function is defined as

$$\text{weighting function} = \frac{\partial U}{\partial \gamma} = -\tau e^{-\gamma\tau}\sin\tau \tag{14}$$

It should be noted that the assumption of

$$\gamma^2 \ll 1.0 \tag{15}$$

has been made and will be used in all subsequent calculations. The minimization procedure is

$$\int_0^{2\pi} \eta \frac{\partial U}{\partial \gamma}\,\mathrm{d}\tau = 0 \tag{16}$$

Substituting Eqs. (13), (14), and (15) into Eq. (16), one obtains a transcendental equation for ϵ and γ as

$$\epsilon = \frac{1 - e^{-4\pi\gamma}}{\frac{8}{9} + e^{-3\pi\gamma}\left[2\left(\frac{8}{9} - 6\pi\gamma\right) + e^{-3\pi\gamma}\left(\frac{8}{9} - 12\pi\gamma\right)\right]} \tag{17}$$

Figure 4 shows the result of Eq. (17). It is clear now that the assumption postulated in Eq. (15) is indeed valid. For a wide range of ϵ, γ remains a small value. Now the solution to Eq. (9) is approximated as

$$U(\tau) = e^{-\gamma\tau}\sin\tau$$
$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = e^{-\gamma\tau}(\cos\tau - \gamma\sin\tau) \tag{18}$$

The equivalent linear approximation solution is plotted along with the exact solution in the Fig. 3 phase-plane diagram for $\epsilon = 0.2$ and corresponding $\gamma = 0.028$. The approximate solution is of the decay sinusoidal type, the decay $e^{-\gamma\tau}$ being of some importance because it is the envelope of each peak. Figure 5 shows this decay envelope together with the exact time history. The approximate envelope accurately describes the decay nature of the peaks.

Next, the energy required to generate in-plane tension is defined. To generate strain in a piezoelectric strain transducer, a certain amount of electric power is needed. It is reasonable to say that the electric power required is proportional to the amount of tension generated regardless of positive tension or negative compression. However, the string will be under net tension all the time because of the pretension T_0. Then the energy required for the control device will be proportional to the absolute value of time-varying tension integrated over the operating time. The efficiency factor α should be defined as the ratio of transducer energy required per one cycle of motion to the energy reduction per cycle of vibration. However, it is assumed that the vibration amplitude reduction is proportional to the energy reduction within a specified range. Therefore,

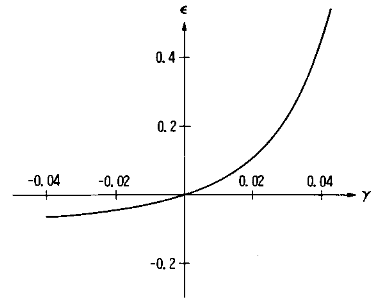

Fig. 4 Tension coefficient ϵ and linear damping τ relationship.

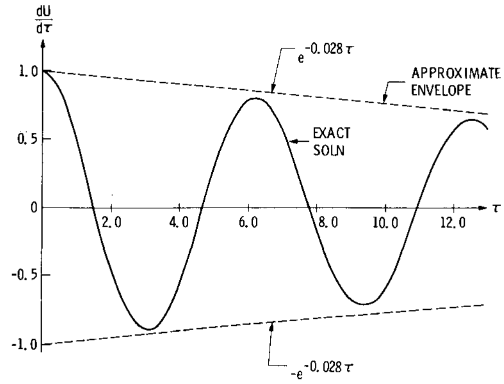

Fig. 5 Approximate decay envelope and exact time history.

$$\alpha = \frac{\text{Energy per Cycle}}{\text{Amplitude Reduction per Cycle}} = \frac{\int_0^{2\pi}|\Delta\tilde{T}|\mathrm{d}\tau}{1 - e^{-\gamma(2\pi)}} \tag{19}$$

Since the nondimensional time-varying tension $\Delta\tilde{T}$ can be defined as

$$\Delta\tilde{T} = \epsilon\left(\frac{\mathrm{d}U}{\mathrm{d}\tau} \times U\right)\Big/|U| \tag{20}$$

and

$$|\Delta\tilde{T}| = \left|\epsilon\frac{\mathrm{d}U}{\mathrm{d}\tau}\right| \tag{21}$$

then the efficiency factor α can be obtained by substituting Eqs. (18) and (21) into Eq. (19) as

$$\alpha = 2\epsilon e^{-\pi/2\gamma}(1 + e^{-\pi\gamma})/(1 - e^{-2\pi\gamma}) \tag{22}$$

Combining Eqs. (17) and (22) the efficiency factor α can be plotted as a function of the tension coefficient ϵ.

Figure 6 is a plot of the efficiency factor α as a function of ϵ. It is desirable from the energy consumption viewpoint to have a small α, and the figure shows that the smaller time-varying tension is more desirable.

Next, the time factor β, defined as the time required to damp out a given disturbance response, will be obtained as a function of ϵ. The response amplitude will be defined as the decay envelope instead of the transient time history. The relationship is

$$U_{\text{output}} = e^{-\gamma\cdot\beta} \tag{23}$$

If one uses 1-db amplitude reduction as the given disturbance amplitude to be damped out, by definition, the response

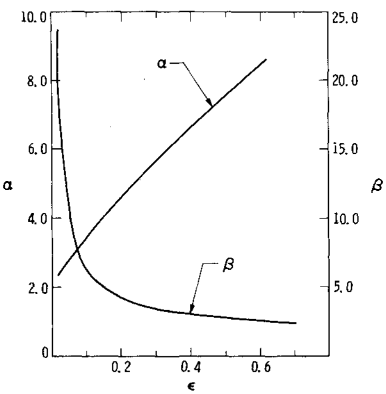

Fig. 6 Efficiency factor α and time factor β as function ε.

output is

$$-1.0 \text{ db} = 20\log_{10}\left(\frac{U_{\text{output}}}{U_{\text{input}}}\right) \tag{24}$$

and

$$U_{\text{output}} = 10^{-0.05} = 0.89125$$

(Note: The initial condition $U_{\text{input}} = 1.0$.)
From Eqs. (23) and (24) one obtains

$$\beta = -(1/\gamma)\ell n(0.89125) \tag{25}$$

Again using Eq. (17), the β can be plotted against ϵ as shown in Fig. 6. With a specified time factor β, which may come from the space structure mission requirement, the magnitude of the time-varying tension ϵ can be determined.

Concluding Remarks

Feasibility of in-plane tension varying for the control of out-of-plane response has been demonstrated. Also, methodologies for obtaining the efficiency factor and the time factor are described. The approach has been carried out on an elastic string with an initial velocity whose spatial variation is similar to its first normal mode. Because of the simplicity of the problem, the essence of the stiffness variation for response control was demonstrated very clearly. Similar results should be obtained if other initial conditions and external excitations are present. For more complicated two- or three-dimensional structural systems, as long as the out-of-plane stiffness is derived from the in-plane tension the concept should work, although the analysis may become very complicated.

For those structures with their own intrinsic out-of-plane bending and torsional stiffness, the in-plane tension control concept does not work. However, the concept of stiffness varying should still be valid. The time-varying stiffness can be accomplished either by the time-varying boundary conditions, which certainly affect the system stiffness properties, or by engaging and/or disengaging some of the structural components. As long as the time-varying part of the stiffness property is a function of the sensor measured response velocity, some active damping effect can be realized. The analysis will be extremely complicated not only because of the amount of stiffness to be varied but also its distribution. Also, feasibility is dependent on hardware design, wherein size and weight must be much more favorable than the conventional jets and rockets in order to make this concept attractive.

Acknowledgments

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